

# Compressed Sensing and HYPR

Julia Velikina, PhD

Department of Medical Physics, University of Wisconsin – Madison

## Introduction

The desire to achieve high spatial and/or temporal resolution in MRI coupled with limited scan time has led to the necessity to reconstruct images from incomplete datasets. A number of approaches to accelerate MR imaging have been proposed, including parallel imaging (1-3), UNFOLD (4), *k-t* BLAST/SENSE (5) and, more recently, compressed sensing (6-8) and HYPR (9,10). We will discuss several different to constrained image reconstruction from incomplete data using both theoretical models and image-specific assumptions.

## Regularized Image Reconstruction

Mathematical concept of a *norm* is one of the key elements in reconstruction algorithms, especially, the  $l_p$  norm defined by

$$\|x\|_p = \left( \sum |x_k|^p \right)^{\frac{1}{p}}.$$

Of particular interest in image reconstructions are  $l_2$ ,  $l_1$ , and  $l_0$  norms (although the last one technically is not a norm as it does not satisfy all the necessary axioms).

MRI signal equation can be represented in the following form:

$$Ef = b, \tag{1}$$

where  $E$  is the encoding matrix containing Fourier encoding terms and, generally, coil sensitivity values,  $f$  is the image vector, and  $b$  is the vector of measured data from all coil receivers. When the number of elements in  $f$ , that is, the number of image pixels, is greater than the number of rows in  $E$ , that is, the number of acquired data points, the linear system in Eq. [1] becomes underdetermined and has an infinite number of possible solutions. The simplest way to isolate a single solution is to minimize  $l_2$  norm of the residue, that is, to solve the following problem:

$$f = \arg \min \|Ef - b\|_2.$$

If  $E$  incorporates coil sensitivity values, then such minimization corresponds to the simplest formulation of parallel imaging (SENSE) approach (1). However, decreased acquisition time and noise amplification (g-factor) lead to increased noise level in the reconstructed imaging.

*Tikhonov Regularization.* One of the ways to solve this problem is to apply Tikhonov regularization (11,12) or the regularization by  $l_2$  norm:

$$f = \arg \min \|Bf - b\|_2^2 + \lambda \|f\|_2^2.$$

where  $\lambda$  is the regularizing parameter that provides a balance between the level of the noise in the reconstructed image and the level of residual artifacts. Improvement in SNR level achieved for higher values of  $\lambda$ , in general, leads to resolution loss and residual aliasing.

*Compressed Sensing.* Recently, a novel mathematical theory has been developed (6) that states that *sparse* images (i.e., images with a relatively small number of pixels containing relevant information) can be accurately reconstructed from undersampled datasets, provided the encoding matrix  $B$  satisfies certain conditions. Ideally, the sparsity of an image is measured by its  $l_0$  norm that counts the number of non-zero pixels. Therefore, if we know in advance that the underlying image is expected to be sparse (as is the case, for example, in MR angiography), then the image may be obtained as

$$f = \arg \min \|Bf - b\|_2^2 + \lambda \|f\|_0.$$

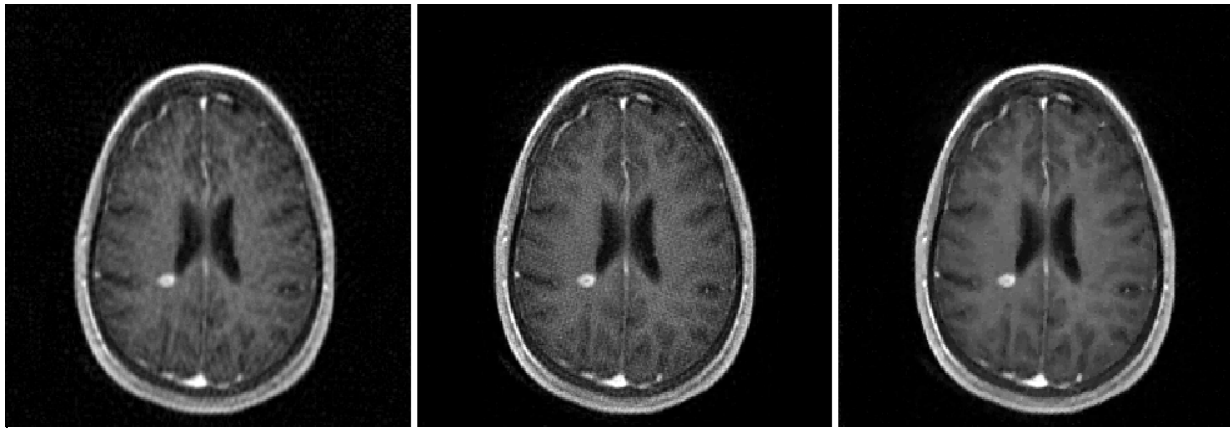
The problem with this formulation is that while it allows for accurate reconstruction of sparse images, the minimization possesses combinatorial complexity, so its practical implementation is infeasible. However, the compressed sensing theory proves that, under certain conditions, the solution of  $l_0$  minimization problem is equivalent to the solution of  $l_1$  minimization problem, i.e. we can solve the following problem:

$$f = \arg \min \|Bf - b\|_2^2 + \lambda \|f\|_1.$$

There are a number of computationally efficient ways to implement  $l_1$  minimization in practice, which made compressed sensing ideas attractive to accelerated MR imaging (7,8).

The admissible acceleration factor is analytically related to the sparsity level of the signal. Higher level of undersampling leads to artifacts in the reconstructed images. This often poses a problem in rapid imaging, since even intrinsically sparse angiographic images may not possess the level of sparsity necessary to support the high acceleration factors desirable in some applications. However, image sparsity can be enhanced either by an application of a sparsifying transform such as an image gradient or a wavelet transform, or by subtracting a prior image estimate (13), or by both. Moreover, several regularizing terms may be used to provide a better reconstruction. Therefore, in its most general formulation compressed sensing solves the following minimization problem:

$$f = \arg \min \left( \|Bf - b\|_2^2 + \sum_k \lambda_k \|W_k(f - f_{0,k})\|_1 \right).$$



**Figure 1.** Reconstruction of an image from an undersampled dataset (acceleration factor 4) using  $\ell_2$  regularization (left),  $\ell_1$  regularization of the image itself (center), and  $\ell_1$  regularization of the image gradient (right).

Here,  $\Psi_k$  are sparsifying transforms and  $f_{0,k}$  are corresponding prior image estimates that may be obtained in a number of ways, for example, from a prior scan, or from more densely sampled low frequencies, or from temporally averaging a time series. Images in Fig. 1 compare the effects of different ways to regularize reconstruction.

We will discuss both theoretical requirements of compressed sensing and some aspects of its practical implementation.

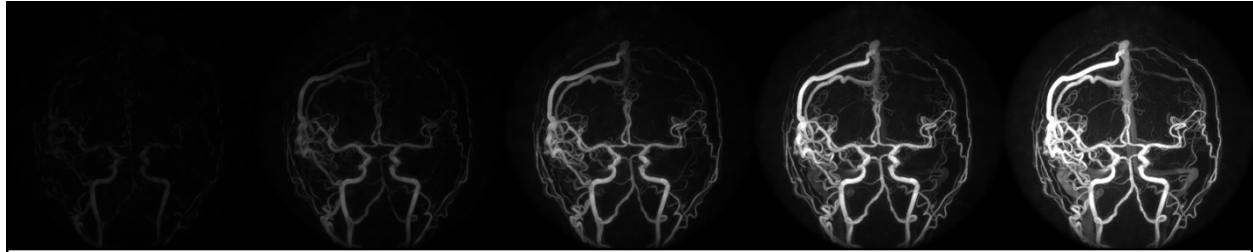
## HYPR

The HighY construct backPRojection (HYPR) method belongs to another family of constrained reconstruction algorithms, which use a multiplicative constraint by a prior image. HYPR reconstruction is usually applied to serial imaging, such as time-resolved imaging or diffusion tensor imaging. A HYPR image is obtained as

$$H = W \cdot C,$$

where  $C$  is the prior image estimate and  $W$  is a weighting image. The prior image estimate,  $C$ , also called the composite image, is usually obtained from averaging all or a subset of the data collected during the exam. The quality of the composite image largely determines spatial resolution and SNR of the individual HYPR frames. The main distinction between different algorithms in the HYPR family lies in the way the weighting images are formed. The original HYPR algorithm (9) and its modification (14) use unfiltered backprojection, and therefore are tailored specifically to radial acquisition. The subsequently developed HYPR LR algorithm (10) relies on  $k$ -space filtering to form the weighting images and can be applicable to any sampling trajectory. Another advantage of HYPR LR is that it reduces signal cross-talk between spatially adjacent objects with different time courses, such as, for example, an artery and a vein. This property allows for the use of composite images collected over a longer period of time and, thus, having higher SNR, which is then transferred to individual HYPR frames. Another possibility is to acquire a composite image in a separate scan, as was done in the HYPR Flow technique

(15). Images in Fig. 2 illustrate contrast arrival in an AVM patient using the HYPR Flow technique.



**Figure 2.** HYPR Flow reconstructed maximum intensity projections of 3D time frames showing contrast arrival in an AVM patient.

Both HYPR and HYPR LR algorithms are approximate image reconstruction techniques. We will discuss the dependence of the reconstruction error and performance of the algorithms on image sparsity and spatio-temporal correlation of the images in the series. We will also discuss several iterative HYPR techniques (16-18) that were developed with the aim of improving the accuracy of the reconstruction.

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